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Research Article

Fuzzy Variable Order Sliding Mode Control of Nonlinear MIMO System

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***Abstract*— This study presents the design of a fuzzy variable-order sliding mode controller for a twin rotor multi-input multi-output system (TRMS). The primary objective is to adaptively modify the sliding surface of the sliding mode controller through variable-order design utilizing fuzzy logic, while investigating the performance impact of the proposed control methodology on MIMO system behavior. The sliding surface degree exhibits varying effects on system performance depending on the error magnitude. Both the gain and order of the sliding surface significantly influence overshoot and reaching time characteristics. When the sliding surface order is reduced, the system experiences increased rise time while demonstrating decreased overshoot. Based on these observations, system performance can be enhanced by adaptively adjusting both the gain and order of the sliding surface in response to error and error derivative signals. Examination of simulation and experimental results demonstrates that this methodology improves key performance indices, including overshoot, rise time, and settling time.**

Index Terms— **Variable Order Systems, Fuzzy Logic Systems, Variable Order Sliding Mode Control.**

I. INTRODUCTION

Variable order systems are systems are mathematical frameworks founded on fractional calculus principles, wherein the system order is expressed as a time-dependent function. Ross and Samko pioneered the conceptualization of system order as a bounded time-varying function in such systems, implementing the Riemann–Liouville integral approach for their formulation [1–3]. Subsequent research in this field has predominantly focused on the modeling and mathematical analysis of variable-order systems. Within this body of work, Sierociuk [4] developed an analog representation of the variable-order derivative, while Lorenzo and Hartley extended variable-order systems to dynamic system applications employing Grünwald–Letnikov and Liouville methodologies based on the Mittag-Leffler function [5]. Tavares et al. focused on the numerical solution of the variable-order Caputo derivative [6]. Zhao and his colleagues developed an algorithm for variable-order systems using a second-order approximation method [7]. The first engineering applications of variable-order systems were implemented in control systems [8–11]. These studies examined the influence of controller order on system performance and implemented parameter adjustment strategies based on system error feedback. The methodologies employed in these works encompassed variable-order PID controllers and extremum seeking techniques.

Unlike other studies, the current research models the sliding surface of a variable-order sliding mode controller (SMC) with a variable order. SMC is recognized for its strong robustness in the presence of external disturbances and system parameter variations. In the literature, this control method has been successfully applied to various real systems such as robotic systems [12], power electronics [13,14], and motor control applications [15,16], demonstrating its effectiveness under such uncertainties. Additionally, studies have shown that SMCs designed using fractional-order models can further improve control performance.

For example, Yiang et al. demonstrated through simulation and practical studies that their fractional-order SMC design for anti-lock braking systems achieved rapid convergence to the desired sliding surface despite system uncertainties [17]. Ebrahimkani, in his simulation study involving doubly-fed induction generators used in wind turbines, showed that the fractional-order SMC improved system efficiency and robustness [18]. In these and similar studies, the chattering problem encountered in SMCs has been a key focus, and many proposed methods have shown success in reducing chattering. It has been demonstrated that intelligent system-based methods are particularly effective in mitigating this issue. Among these, Wang et al. used a fuzzy logic approach to reduce chattering in SMCs [19]. Safa et al. addressed the problem in micro/nano positioning systems [20], while Shaer et al. succeeded in minimizing chattering in active suspension systems [21]. Consistent observations indicate that chattering is significantly reduced when the sliding surface or associated control input is determined through fuzzy logic or other intelligent computational techniques [21–27]. In intelligent system-based adaptive SMCs, the slope of the sliding surface is typically adapted based on system error. However, the present study introduces a novel approach wherein both the slope and order of the sliding surface are adaptively modified in response to system error dynamics. This methodology is designed to enhance performance characteristics in nonlinear system applications. The performance of the designed controller was assessed using a Twin Rotor MIMO System (TRMS) test platform. Helicopters, known as rotary-wing aerial vehicles,

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serve as excellent examples of Multi-Input Multi-Output (MIMO) systems due to the numerous uncertainties affecting system performance. Among rotary-wing systems, TRMSs are widely used to test control algorithms. Given that the motion of both rotors exerts direct and indirect influences on pitch and yaw angular dynamics, these systems are categorized as MIMO platforms. The inherent coupling effects between the dual rotors, combined with the system's nonlinear characteristics, present significant challenges in control system [31,32].

Many studies have attempted to address these challenges through modeling and control design. Controller modeling for TRMSs has been conducted under various approaches such as modeling with hybrit neural networks [33], dynamic modeling of hovering TRMS [34], and ANFIS modeling using particle swarm optimization and RLS [35]. Notable studies have also focused on pitch and yaw angle control. Various methods have been developed to achieve robust and stable control. For instance, TRMS performance has been optimized using a GA-optimized PI controller [31], a novel fuzzy SMC method [32], and a fuzzy LQR controller [36]. Additionally, fractional-order control represents another promising control paradigm applied to TRMS platforms

This study implements a fuzzy fractional-order sliding mode controller for TRMS control applications. The primary focus is to investigate the performance impact of sliding surface parameters, adaptively adjusted through fuzzy logic, on nonlinear MIMO system behavior. Pitch and yaw angular variations are analyzed through comprehensive simulation studies and experimental validation. Previous studies have shown that modifying the sliding surface improves control performance in nonlinear systems. As a novel contribution, this study applies a variable sliding surface SMC to the nonlinear MIMO TRMS. Controller performance evaluation was conducted using established metrics including Total Squared Error (TSE), overshoot, and settling time. The simulation was carried out for both classical SMC and the proposed variable-order method. The effect of the variable-order control method was also examined in the experimental setup. The results, including the variations in pitch angle, tail angle, applied voltage, and sliding surface, are presented graphically. A comparative table was created based on insights derived from these graphs.

The paper is organized as follows: Section I provides an overview of the structure of twin-rotor MIMO systems. Section II details the design and stability analysis of the variable-order sliding mode controller. Section III presents the simulation and experimental results, and finally, Section IV discusses the findings and provides the conclusions..

II. TWIN ROTOR MIMO SYSTEMS (TRMS)

Aerial vehicles exhibit highly complex dynamics due to airflow-induced force uncertainties and inherent nonlinear aerodynamic characteristics. Consequently, autonomous control of such platforms presents significant challenges. As a result, these types of vehicles serve as standard benchmark platforms for control system design and controller performance validation. In this investigation, the Twin Rotor Multi Input Multi Output System (TRMS) experimental platform, manufactured by Feedback Instruments, was employed to evaluate the performance of the proposed controller. The system configuration is illustrated in Fig. 1. This system has two rotors: a main rotor, which enables vertical motion, and a tail rotor, which enables horizontal motion, resembling a helicopter model. The mathematical expressions of this system are given between Equations (1)-(11), and as can be seen from these expressions, the system consists of many nonlinear dynamics.

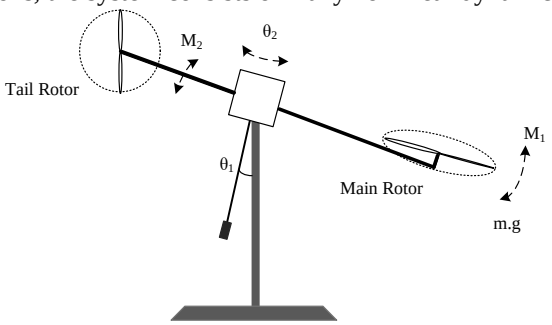


Fig. 1. Twin rotor MIMO prduced by Feedback.

The dynamic equations corresponding to the tail rotor and main rotor motors of the system are given in Equation (1) and Equation (2), respectively.

$$J_1 \frac{d^2\theta_1}{dt^2} = M_1 - M_{FG} - M_{B\theta_1} - M_G \quad (1)$$

$$J_2 \frac{d^2\theta_2}{dt^2} = M_2 - M_{B\theta_2} - M_R \quad (2)$$

where, the angles Θ_1 and Θ_2 are varied on the main rotor and tail rotor sides, respectively. The torque produced by the motor used in the TRMS has a nonlinear characteristic, and is given in Equations (3) and (4).

$$M_1 = a_1.{\tau_1}^2 + b_1.\tau_1 \quad (3)$$

$$M_2 = a_2.{\tau_2}^2 + b_2.\tau_2 \quad (4)$$

These values depend on the voltage applied to the motor, and the expressions τ_1 and$\tau_2$used in the equation are given below.

$$\tau_1 = \frac{k_1}{T_{11}s+T_{10}}u_1 \quad (5)$$

$$\tau_2 = \frac{k_2}{T_{21}s+T_{20}}u_2 \quad (6)$$

where, u_1 and u_2 represent the voltages applied to the main and tail motors, respectively. The other coefficients k_1, k_2, T_{11} and T_{10} are motor parameters. The forces caused by friction within the system are expressed as the friction torques of the tail and main rotors, respectively, as follows:

$$M_{B\theta_1} = B_{1\theta_1}.\dot{\theta}_1 + B_{2\theta_1}.sign(\dot{\theta}_1) \quad (7)$$

$$M_{B\theta_2} = B_{1\theta_2}.\dot{\theta}_2 + B_{2\theta_2}.sign(\dot{\theta}_2) \quad (8)$$

where, $B_{1\theta_2}$and $B_{2\theta_2}$are friction coefficients. In the TRMS system, the gyroscopic moment, gravitational moment, and cross-reaction moment are also present and are given respectively in Equations (9)-(11).

$$M_G = K_{GY}.M_1.\dot{\theta}_2.cos(\theta_1) \quad (9)$$

$$M_{FG} = K_{GY}.M_1.\dot{\theta}_2.cos(\theta_1)\,sin(\theta_1) \quad (10)$$

$$M_R = \frac{k_c(T_0 s+1)}{T_p s+1}\tau_1 \quad (11)$$

K_{GY}, T_0, T_p are the system parameters. All parameters related to the TRMS system are provided in the Appendix [34]..

III. VARIABLE ORDER SLIDING MODE CONTROLLER DESIGN

A. *Variable Order Systems and Controller*

In recent years, many solution methods have been proposed for fractional variable-order differential equations. Among these methods, three fundamental approaches have been identified: Grünwald-Letnikov, Caputo, and Riemann-Liouville. (Sierociuk et al., 2015; Tavares et al., 2016; Zhao et al.,

2015). In the numerical analysis of fractional variable-order systems, the Grünwald-Letnikov method is used.

Definition 1: The Type-I method of the fractional variable-order derivative based on the Grünwald-Letnikov approach is presented below.;

$$ {}_aD_t^{\alpha(t)} f(t) = \lim_{h\to 0} \sum_{k=0}^{[(t-a)/h]} \frac{(-1)^k}{h^{\alpha(t)}} \binom{\alpha(t)}{k} f(t-kh) \quad (12)$$

Definition 2: The Type-II method of the fractional variable-order derivative based on the Grünwald-Letnikov approach is presented below.

$$ {}_aD_t^{\alpha(t)} f(t) = \lim_{h\to 0} \sum_{k=0}^{[(t-a)/h]} \frac{(-1)^k}{h^{\alpha(t-kh)}} \binom{\alpha(t-kh)}{k} f(t-kh) \quad (13)$$

Definition 3: The Type-III method of the fractional variable-order derivative based on the Grünwald-Letnikov approach is presented below.

$$ {}_aD_t^{\alpha(t)} f(t) = \lim_{h\to 0} \sum_{k=0}^{[(t-a)/h]} \frac{(-1)^k}{h^{\alpha(kh)}} \binom{\alpha(kh)}{k} f(t-kh) \quad (14)$$

Similarly, in the Riemann-Liouville method, there are also two different approaches. These are:

Definition 4: The Type-I left and right approaches of the Riemann-Liouville method are given below, respectively:

$$ {}_aD_t^{\alpha(t)} f(t) = \frac{1}{\Gamma(1-\alpha(t))} \frac{d}{dt} \int_a^t (t-\tau)^{\alpha(t)} f(\tau)\, d\tau \quad (15)$$

ve

$$ {}_aD_t^{\alpha(t)} f(t) = \frac{-1}{\Gamma(1-\alpha(t))} \frac{d}{dt} \int_t^b (\tau-t)^{-\alpha(t)} f(\tau)\, d\tau \quad (16)$$

Definition 5: The Type-II left and right approaches of the Riemann-Liouville method are given below, respectively;

$$ {}_aD_t^{\alpha(t)} f(t) = \frac{d}{dt} \left(\frac{1}{\Gamma(1-\alpha(t))} \int_a^t (t-\tau)^{\alpha(t)} f(\tau)\, d\tau \right) \quad (17)$$

ve

$$ {}_aD_t^{\alpha(t)} f(t) = \frac{d}{dt} \left(\frac{-1}{\Gamma(1-\alpha(t))} \int_t^b (\tau-t)^{-\alpha(t)} f(\tau)\, d\tau \right) \quad (18)$$

B. *Variable Order Sliding Mode Controller*

As with fractional-order systems, variable-order controllers were initially introduced to control system applications within engineering disciplines. These controllers are advantageous due to their extended stability regions and superior performance characteristics.

In classical sliding mode control methods, the sliding surface is selected based on the variation of the error, as given below. (Van Chi, 2015).

$$ S(t) = \Gamma e(t) + \dot{e}(t) \quad (19)$$

Accordingly, the system's error function $e(t)$ is defined as;

$$ e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} \theta_{1_ref}(t) - \theta_1(t) \\ \theta_{2_ref}(t) - \theta_2(t) \end{bmatrix}$$

$$ \Gamma = \begin{bmatrix} \kappa & 0 \\ 0 & \kappa \end{bmatrix}, \kappa > 0 \quad (20)$$

where κ denotes a positive constant.

In variable-order sliding mode controllers, the controller's sliding surface can be chosen as variable-order. The variable-order sliding surface is defined as given in Equation 21.

$$ S(t) = \Gamma e(t) + D^{\alpha(t)} e(t) \quad (21)$$

Where $D^{\alpha(t)}$ denotes the fractional-order derivative or integral operation. According to the Lyapunov stability theorem;

$$ V(t) = \frac{1}{2} S^T(t)\, S(t) > 0$$

$$ \dot{V}(t) = S^T(t) \frac{dS(t)}{dt} < 0 \quad (22)$$

To satisfy the condition, the criteria given in Equations 23 and 24 must be met.

$$\begin{aligned} \dot{V}(t) &= \left[\begin{bmatrix} \kappa & 0 \\ 0 & \kappa \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} + D^{\alpha(t)} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \right]^T \left[\begin{bmatrix} \kappa & 0 \\ 0 & \kappa \end{bmatrix} \begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} \right. \\ &\quad \left. + D^{1+\alpha(t)} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \right] < 0 \\ &= \left[[\kappa e_1(t) \quad \kappa e_2(t)] + D^{\alpha(t)} [\kappa e_1(t) \quad \kappa e_2(t)] \right] \left[\begin{bmatrix} \kappa \dot{e}_1(t) \\ \kappa \dot{e}_2(t) \end{bmatrix} \right. \\ &\quad \left. + D^{1+\alpha(t)} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \right] \\ &= \left[[\kappa e_1(t) \quad \kappa e_2(t)] + D^{\alpha(t)} [\kappa e_1(t) \quad \kappa e_2(t)] \right] \left[\begin{bmatrix} \kappa \dot{e}_1(t) \\ \kappa \dot{e}_2(t) \end{bmatrix} \right. \\ &\quad \left. + D^{1+\alpha(t)} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \right] \\ &= \kappa^2 e_1(t)\, \dot{e}_1(t) + \kappa^2 e_2(t)\, \dot{e}_2(t) + D^{1+\alpha(t)} \kappa {e_1}^2(t) \\ &\quad + D^{1+\alpha(t)} \kappa {e_2}^2(t) + \cdots \\ &\ldots D^{\alpha(t)} \kappa^2 e_1(t)\, \dot{e}_1(t) + D^{\alpha(t)} \kappa^2 e_2(t)\, \dot{e}_2(t) + \\ &D^{1+2\alpha(t)} \kappa {e_1}^2(t) + D^{1+2\alpha(t)} \kappa {e_2}^2(t) \end{aligned} \quad (23)$$

$$ u(t) = K\, sgn(S)$$

C. *Effect of the Sliding Mode Parameter on Controller Performance*

In the sliding mode control method, the κ parameter and the order of the sliding surface have been chosen as the transfer function given in Equation 24 to demonstrate their effects on control performance.

$$ G(s) = \frac{1}{s^2+2s+1} \quad (24)$$

κ by examining the system response for different values of the SM parameter and angle α, the variation of the total squared error, representing system performance and given by the expression in Equation 25, is shown respectively in Fig. 2. and 3. In Fig. 2., it is observed that as the system order increases, the convergence rate increases; however, the oscillation also becomes more pronounced. In Fig. 3., the effect of the gain coefficient κ on overshoot and settling time can be observed.

$$ ISE = \int |e(t)|^2 dt \quad (25)$$

Additionally, the effects of the controller on performance parameters such as overshoot and settling time are presented in Table 1. Similarly, the impact of variations in the κ parameter is shown in Fig. 3. and Table 2. In Table 1, it is observed that as the fractional order increases, the overshoot decreases; however, the settling time increases. In Table 3, it is observed that as the SM coefficient increases, the overshoot increases, while the settling time decreases.

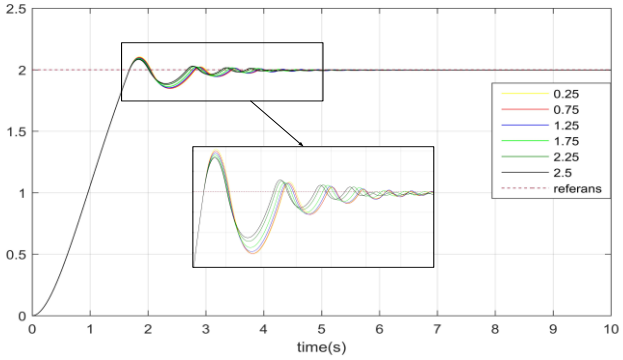


Fig. 2. Effect of sliding surfaces with different orders on system performance

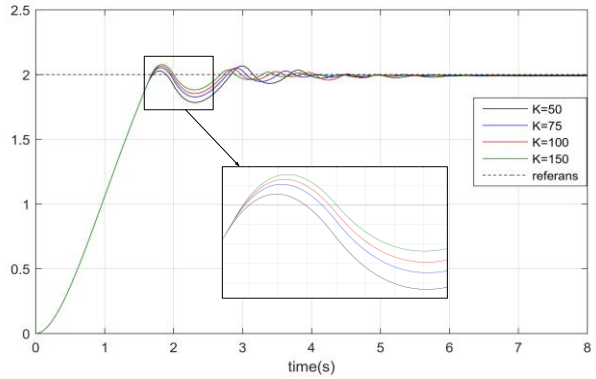


Fig. 3. κ The effect of the variation in the parameter on performance.

Table I. The effects of the fractional order (α) on overshoot and settling time

Fractional order of controller (α)	Overshoot (%)	Settling Time in Control System
0.25	5.22	3.698
0.75	4.98	3.753
1.25	4.75	3.838
1.75	4.52	3.893
2.25	4.29	3.910
2.5	4.17	3.919

Table 1. κ The effects of the SMC parameter on overshoot and settling time

κ	Overshoot (%)	Settling Time in Control System
50	3.24	6.061
75	3.31	5.243
100	3.61	5.213
150	3.86	5.106

As demonstrated by the performance indicators presented in Fig. 1. and 2., and Tables 1 and 2, variations in the fractional order α and parameter κ significantly influence overshoot and settling time characteristics. Adaptive adjustment of these parameters based on error feedback is expected to enhance overall system performance.

D. Fuzzy Adaptive Fractional Order Control

As observed in the example given in the previous section, the variability of the κ parameter and the fractional order provides an opportunity to influence the performance of the controlled system positively. However, determining appropriate values for these two parameters using classical approaches and a single mathematical model is almost impossible. Therefore, in this study, these parameters are determined with the help of fuzzy logic, which can be modeled based on expert knowledge and the system dynamics.

As demonstrated in the preceding example, increasing the fractional order reduces overshoot while simultaneously increasing settling time. This trade-off phenomenon similarly applies to the κ parameter. The fuzzy logic system employed in the present study utilizes a rule base and membership functions, developed from expert knowledge, to adaptively adjust both parameters in response to the resulting error. In other words, the κ parameter and the fractional order are determined by a fuzzy logic structure with two inputs and two outputs based on the error. The block diagram of our system is shown in Fig. 4.

The membership functions used during the fuzzification and defuzzification stages of the fuzzy system developed for our study are presented in Fig. 4 and 5, respectively, and the system's rule table is given in Table 3.

As shown in the previous example, increasing the fractional order reduces the overshoot but increases the settling time. The same dilemma applies to the κ parameter. In the fuzzy logic system used in this study, a rule base and membership functions developed from expert experience were employed to adjust both parameters according to the error resulting from their variations. In other words, the κ parameter and fractional order were determined by a two-input, two-output fuzzy logic structure based on the error. The block diagram of our system is shown in Fig. 4.

The membership functions used in the fuzzification and defuzzification stages of the fuzzy system developed for this study are presented in Fig. 4 and 5, respectively, and the system's rule table is given in Table 3.

IV. RESULTS

The designed controller to test the control system's performance was applied to the multi-input multi-output (MIMO) TRMS system shown in Fig. 6, and the obtained simulation and experimental results were compared with the classical SMC. The parameters of the TRMS system are provided in the appendix section at the end of the study. The results section of our study is divided into two parts: simulation and experimental.

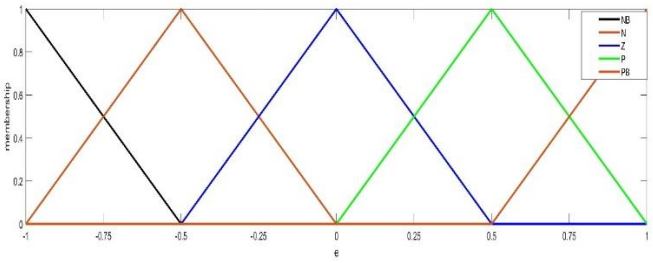


Fig. 4. Membership function of the error

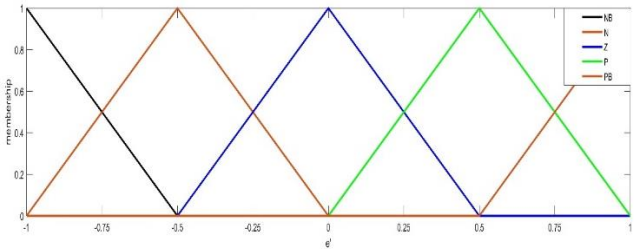


Fig. 5. Membership function derivative of the error

Table 2. Rule base designed for the target controller

$e(t)$ / $\dot{e}(t)$	NB	N	S	P	PB
NB	PB	PB	P	S	S
N	PB	PB	P	S	S
S	PB	PB	S	S	S
P	PB	PB	S	S	S
PB	PB	PB	S	S	S

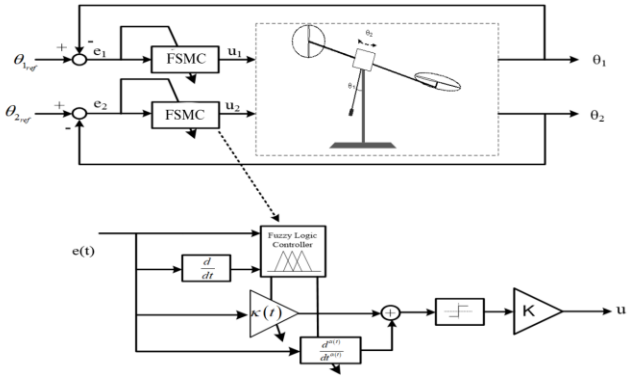


Fig. 6. The Proposed controller block diagram

A. *Simulaiton Results*

In this section, the controller designed for the TRMS system is compared with classical sliding mode controllers based on performance criteria including overshoot, settling time, and total squared error. The system's lateral angle and pitch angle are denoted as θ_1 and θ_2, respectively, while the voltages applied to the tail rotor are symbolized as u_1 and u_2.

The initial reference values for the pitch angle and lateral angle (θ_{1_ref} and θ_{2_ref}) are set to zero. To evaluate the performance of the applied control method, the classical sliding mode control (SMC) method was first implemented. The sliding mode behavior for the pitch angle using the classical SMC is presented in Fig. 7, showing noticeable oscillations in the pitch angle variation. These oscillations are also observed in the input voltage applied under classical sliding mode control, as seen in Fig. 9. Similarly, oscillations in the tail rotor angle and corresponding chattering issues in the applied voltage are observed. Furthermore, the sliding surface approaches zero over time, as illustrated in Fig. 10.

As shown in Fig. 11, the pitch angle variation obtained with the designed controller reaches a steady-state stable condition within approximately 18 seconds, with an overshoot close to 0 radians. Performance analysis of the pitch angle demonstrates superior overshoot and settling time characteristics for the proposed controller relative to the conventional sliding mode controller.

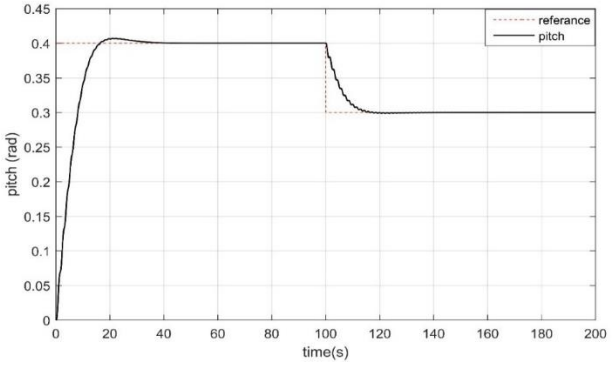


Fig. 7. Pitch angle control using classical sliding mode control

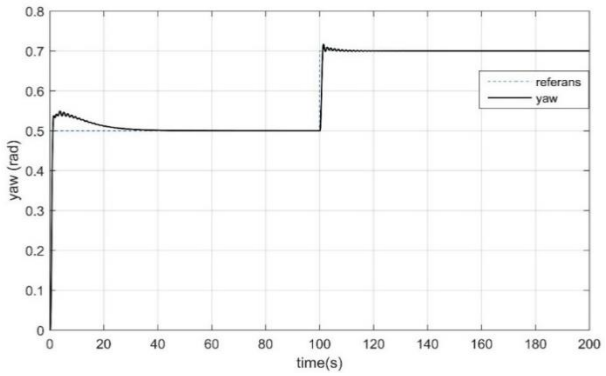


Fig. 8. Pitch angle control with classical sliding mode control

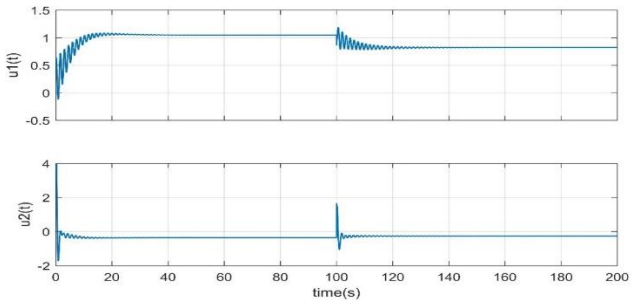


Fig. 9. Voltages applied to the main rotor and tail rotor for pitch angle control

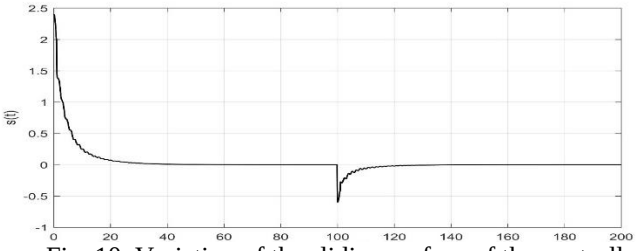


Fig. 10. Variation of the sliding surface of the controller

These results demonstrate that the proposed controller exhibits superior performance for nonlinear MIMO systems, as exemplified by the TRMS platform, surpassing conventional control approaches Furthermore, Fig. 11 and 12 illustrate a noticeable reduction in chattering when using the designed controller compared to the classical sliding mode method. The control result for the yaw angle obtained with the designed controller is presented in Fig. 12. Rise time, overshoot, and settling time for the yaw angle can be observed in this figure. The improvement in this parameter is relatively less compared to the pitch angle. While performance in terms of rise time has increased, an increase in settling time has also been observed. No improvement in overshoot was detected. The voltages applied to the motors are shown in Fig. 13, where the applied voltage and its effect on the angle variation can be observed. Similarly, the variation of the sliding surface is illustrated in Fig. 14.

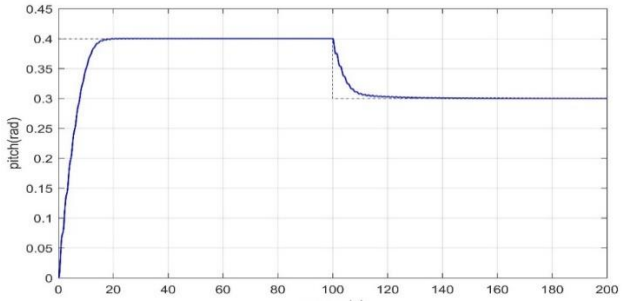


Fig. 11. Control performance of the designed controller for pitch angle

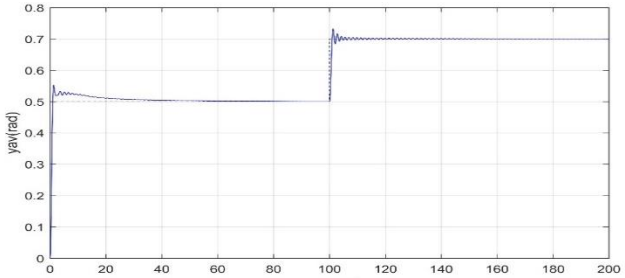


Fig. 12. Pitch angle control performance of the designed controller

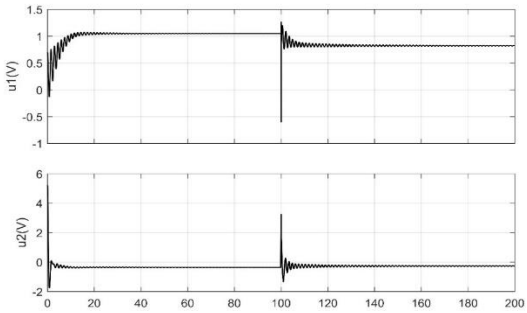


Fig. 13. Voltages applied to the main and tail rotors for pitch angle control

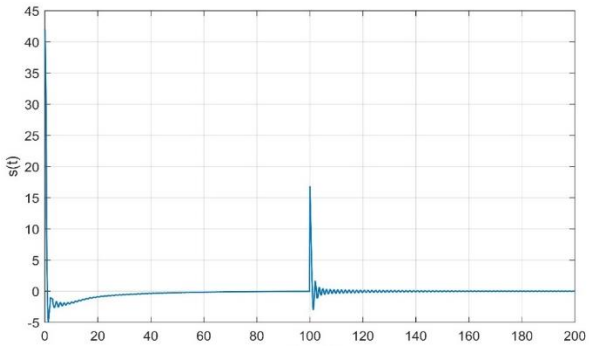


Fig. 14. Sliding surface of the controller

The Integral of Squared Error (ISE) has been used to evaluate the performance of the controller. According to the results, it is observed in Table 4 that the designed controller yields more successful results for nonlinear MIMO systems compared to the other controllers

Table 3. Table comparing the performance of classical SMC and variable-order SMC

Controller	Overshoot		Settling Time in Control System		ISE	
	Pitch (%)	Yav (%)	Pitch (s)	Yav (s)	Pitch	Yav
Classical SMC	1.75	9	36	30	0.547	0.133
Variable-Order SMC	0.01	9	20	30	0.307	0.133

B. Experimental Results

In this study, the experimental results were obtained using the experimental setup shown in Fig. 15. The experimental studies were carried out on a TRMS produced by Feedback The TRMS consists of a main horizontal rotor and a vertical tail rotor mounted on a beam that can freely rotate in pitch and yaw axes. In this setup, the control system for measuring and controlling the pitch and yaw angles was designed using Matlab/Simulink software on a computer. The obtained results were also plotted via the Simulink program.

In the designed control method, it is observed that the variation of the pitch angle is stable, as shown in Fig. 16 and 17, and resembles the change obtained in the simulation. Unlike the simulation, the experimental results show an increase in chattering in the controller performance, as well as slight increases in overshoot and settling time. These inevitable increases are considered to be caused by environmental factors.

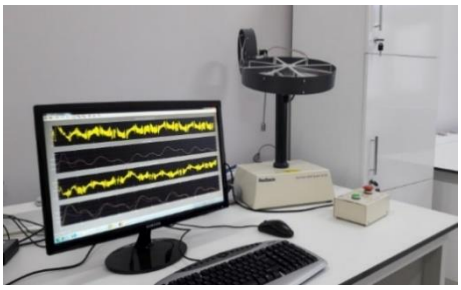


Fig. 15 Experimental test bench

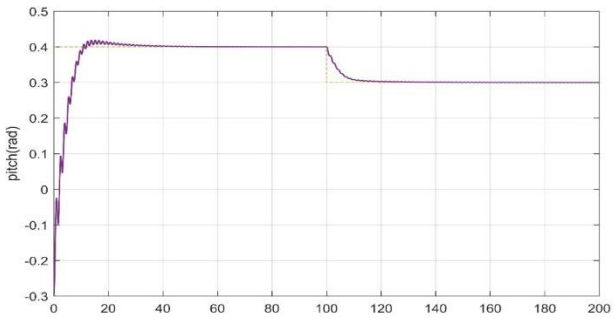


Fig. 16. The variation of the angle of pitch angle in the designed control method

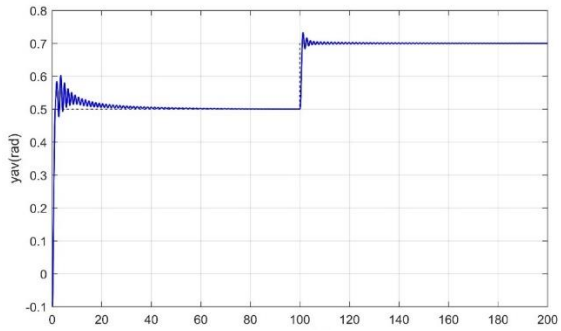


Fig. 17. The variation of the yaw angle in the designed control method

V. CONCLUTİON

This study presents the design of a variable-order fuzzy sliding mode controller for a twin-rotor MIMO system that models dual-rotor helicopter dynamics. The TRMS exhibits complex dynamic characteristics encompassing uncertainties and external disturbances, establishing it as an extensively utilized benchmark platform for control algorithm performance evaluation. Consequently, the proposed variable-order fuzzy sliding mode controller was validated through both simulation studies and experimental implementation.

Controller performance was assessed through comparative analysis with a conventional sliding mode controller using established metrics including overshoot, rise time, settling time, and integral squared error (ISE). Simulation results revealed substantial improvements in overshoot and settling time characteristics, along with approximately 20% enhancement in total squared error performance. However, experimental validation exhibited deviations from simulation outcomes due to unmodeled initial conditions and environmental factors.

Future work will focus on adaptive controller order adjustment, incorporation of system time delay considerations, and a comprehensive investigation of the resulting control performance characteristics. In addition, future research will address robustness against external disturbances, the extension of the proposed approach to other MIMO platforms, and real-time implementation aspects to enhance practical applicability.

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Appendix

The parameters of the TRMS system are given in the table below. [34]

Parameters		Value
J_1	Main rotor moment of inertia),	$6.8\ 10^{-2}$ kg.m^2
J_2	Tail rotor moment of inertia	2.10^{-2} kg.m^2
a_1	Static characteristic parameter	0.0135
b_1	Static characteristic parameter	0.0924
a_2	Static characteristic parameter	0.02
b_2	Static characteristic parameter	0.09
T_G	Gravitational momentum	0.32
$B_{1\Theta1}$	Friction moment parameter	6.10^{-3} N.m.s/rad
$B_{2\Theta1}$	Friction moment parameter	1.10^{-3} N.m.s/rad
$B_{1\Theta2}$	Friction moment parameter	1.10^{-1} N.m.s/rad
$B_{2\Theta2}$	Friction moment parameter	1.10^{-2} N.m.s/rad
K_{GY}	Gyroscopic momentum parameter	0.05 rad/s
k_1	Motor 1 gain	1.1
k_2	Motor 2 gain	0.8
T_{11}	Motor 1 parameter	1.1
T_{10}	Motor 1 parameter	1
T_{21}	Motor 1 parameter	1
T_{20}	Motor 1 parameter	1
T_p	Cross-reaction momentum parameter	2
T_0	Cross-reaction momentum parameter	3.5
k_c	Cross-reaction momentum gain	-0.5
M_1	The torque produced by the Motor 1	
M_2	The torque produced by the Motor 2	

BIOGRAPHIES



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